Supplementary Material for:

Retrieving Jones Matrix from an Imperfect Metasurface Polarizer

Guanqing Zhang[a], Zixian Hu[a], Qichang Ma[a], Jiaming Huang[a], Junhong Deng[b], and Guixin Li[a,c,d,*]

[a]Department of Materials Science and Engineering, Southern University of Science and Technology, Shenzhen 518055, China.

[b]Shenzhen Institute for Quantum Science and Engineering, Southern University of Science and Technology, Shenzhen 518055, China.

[c]Institute for Applied Optics and Precision Engineering, Southern University of Science and Technology, Shenzhen 518055, China

[d]Guangdong Provincial Key Laboratory of Functional Oxide Materials and Devices, Southern University of Science and Technology, Shenzhen 518055, China

1 Experimental Setups

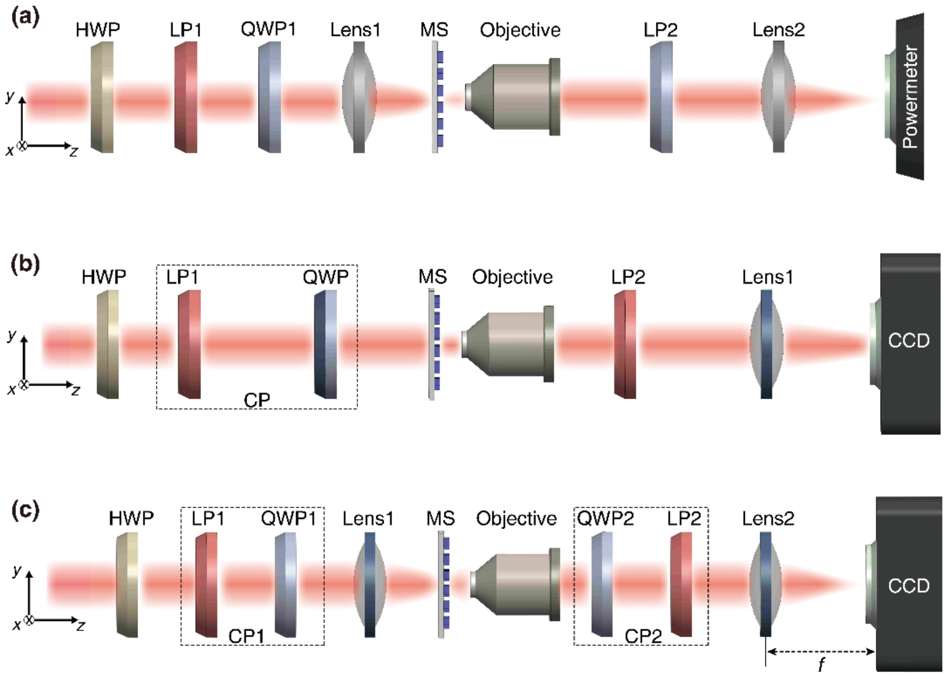

Fig. S1 Experimental setups for characterizing the metasurface polarizers. (a) Setup for measuring the metasurface polarizer's Jones matrix by using a power meter. (b) Setup for characterizing the polarization image encryption properties of the metasurfaces. CP is used to produce the circularly polarized incident light. (c) Setup for capturing the holographic images by using a CCD camera. CP1 and CP2 are used to generate and analyze the circularly polarized light. Half-wave plate: HWP. Linear polarizer: LP. Quarter-wave plate: QWP. Metasurface: MS.

2 Depiction of polarization ellipse of metasurface polarizers

The experimental results for retrieving the Jones matrix of a metasurface is depicted in the Fig. S2(a). The incident light with an averaged power of 100 μW and is focused onto the metasurface polarizers from the substrate side by using the lens L1 with focal length of f = 100 mm. For different incident polarization states (H, 45°, V, 135°, LCP and RCP), the intensity of the transmitted light was measured by rotating the linear analyzer LP2. The power information is recorded at an interval of 10 degrees.

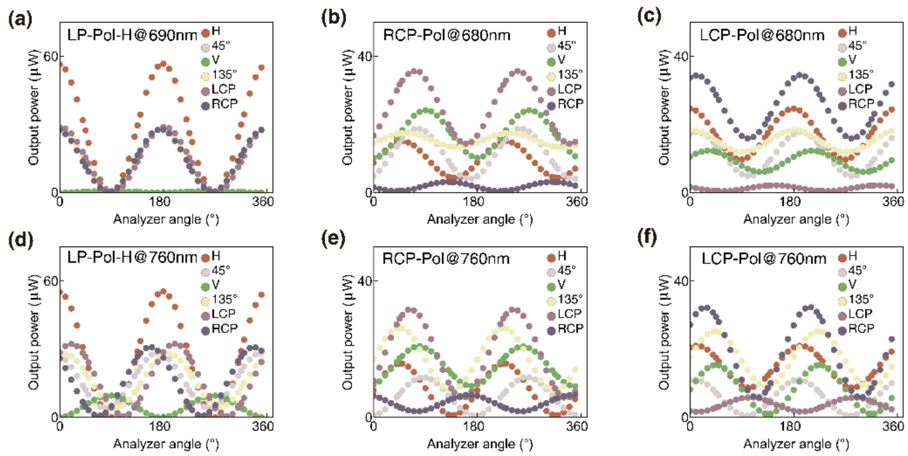

Fig. S2 Experimental results for retrieving the Jones matrix of the linear and circular metasurface polarizers. (a)–(f) Experimental results corresponding to the LP-Pol-H, RCP-Pol and LCP-Pol working at different wavelengths. The six kinds of incident polarization states include H, 45°, V, 135°, LCP and RCP.

The first step in the data analysis process is to plot the intensity as a function of the rotation angle of the linear analyzer. For the six kinds of incident polarization states, the polarization angle dependent transmission power of three metasurface polarizers are shown in Figs. S2(a)–S2(c), respectively. The linear and circular metasurface polarizers exhibit their best performance at the wavelengths of 690 nm and 680 nm, respectively. In comparison, Figs. S2(d)–S2 (f) show the results at $\lambda = 760$ nm which deviates from the optimal wavelengths. It can be found that the measured data exhibits a sine curve. After trigonometric fitting, the maximum I_{max} and minimum I_{min} intensity values of the transmitted light along with their corresponding analyze angles are obtained. The polarization state can be described in the Jones vector form, which requires information of the electric field components in both x and y directions, as well as the phase difference between them. By following the relationship between the intensity and amplitude of the electric field of light, $I = \frac{c\varepsilon_0}{2} E^2 \sim E^2$, we firstly then take the square root of the transmitted power to obtain the relative value of E. Then, the Cartesian coordinate system is transformed to the polar type. The relationship between the analyzer angle θ and the angle δ in polar coordinate is given by $\delta = \tan^{-1}\left(\tan\theta\sqrt{I_{\max}} / \sqrt{I_{\min}}\right)$. Finally, the polarization ellipse which is shown by the point diagram of main text Fig. 3 (right column) can be plotted.

3 Retrieval of the Jones matrix factor

As shown in main text Fig. 1(e), the electric field components along the x and y axes can be determined from the external rectangle of the polarization ellipse. By parametrically fitting the experimental data in the polar coordinates, the equation of the polarization ellipse can be extracted. This allows us to obtain the relative values of the amplitudes of E_{0x} and E_{0y}.

Using the Jones vector, the polarized light can be expressed as:

$$\vec{\mathbf{E}} = \begin{bmatrix} E_x(t) \\ E_y(t) \end{bmatrix} = \begin{bmatrix} E_{0x}e^{i\varphi_x} \\ E_{0y}e^{i\varphi_y} \end{bmatrix} = \begin{bmatrix} E_{0x} \\ E_{0y}e^{i\varepsilon} \end{bmatrix} \tag{1}$$

where ε is the phase difference between E_{ix} and E_{iy}. Alternatively, this phase difference can be determined by using the following equation, which contains parameters of the electric fields:

$$|\varepsilon| = \cos^{-1}[\frac{(E_{0x}{}^2 - E_{0y}{}^2)\tan 2\psi}{2E_{0x}E_{0y}}] \tag{2}$$

where, ψ represents the main axis of the polarization ellipse, which can be obtained from the ellipse's equation. However, since the direction of rotation of polarized light is unknown, only the absolute value of the phase difference can be determined.

To characterize the Jones matrix of the metasurfaces, we used six kinds of output Jones vectors, which correspond to the six incident polarized states (H, $45°$, V, $135°$, LCP and RCP), to retrieve the four complex elements.

$$\begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \begin{bmatrix} E_{ix} \\ E_{iy} \end{bmatrix} = \begin{bmatrix} E_{tx} \\ E_{ty}e^{i|\varepsilon|} \end{bmatrix} \tag{3}$$

The Jones matrix's four complex elements can be obtained by finding the minimum value of the sum of the determinant shown below:

$$\sum_{n=1}^{6}[(E_{ixn}J_{11} + E_{iyn}J_{12} - E_{txn}) + (E_{ixn}J_{21} + E_{iyn}J_{22} - E_{tyn}e^{i|\varepsilon|})] \tag{4}$$

where n corresponds to the six kinds of incident polarization states. When the polynomial takes a minimum value of 0, we obtain the target Jones matrix of the metasurface polarizers. It should be noted that the imperfections of the linear polarizer and wave plates used in the experiment will affect the accuracy of the measurements. To further verify the correctness of the Jones matrix, we also calculated the ellipses by multiplying the incident Jones vector with the retrieved Jones matrix. As shown in Fig. 3 (right column) of the main text, it can be found that the calculated results agree well with the measured ones. As the phase difference ε of the calculated transmitted Jones vector is specific, allowing us to determine the rotation direction of the ellipses.

4 Decomposition of Jones Matrix

Based on Eq. 2 in the main text, the measured Jones matrix of the metasurface polarizers LP-Pol-H, RCP-Pol and LCP-Pol can be described as follows:

$$\begin{bmatrix} J_{HLP11} & J_{HLP12} \\ J_{HLP21} & J_{HLP22} \end{bmatrix} \approx \alpha \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + \beta \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{5}$$

$$\begin{bmatrix} J_{RCP11} & J_{RCP12} \\ J_{RCP21} & J_{RCP22} \end{bmatrix} \approx \frac{\alpha}{2} \begin{bmatrix} i & 1 \\ 1 & -i \end{bmatrix} + \beta \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{6}$$

$$\begin{bmatrix} J_{LCP11} & J_{LCP12} \\ J_{LCP21} & J_{LCP22} \end{bmatrix} \approx \frac{\alpha}{2} \begin{bmatrix} -i & 1 \\ 1 & i \end{bmatrix} + \beta \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{7}$$

where α and β are the complex coefficients of the target Jones matrix and the residual, respectively. The approximate equality symbol indicates that the matrix elements may have small deviations from the ideal ones. To present the data clearly, we define the signal-to-noise ratio as $\kappa = |\alpha / \beta|$. The higher the κ, the closer the performance of the polarizer is to the ideal condition, and vice versa.

The wavelength dependent signal-to-noise spectra are shown in Fig. S3. For the LP-Pol-H metasurface, the optimal polarization conversion performance is observed at wavelength of ~ 690 nm, where the κ is close to 10. Similarly, the circular metasurface polarizers exhibit the optimal performance at the wavelength of 680 nm.

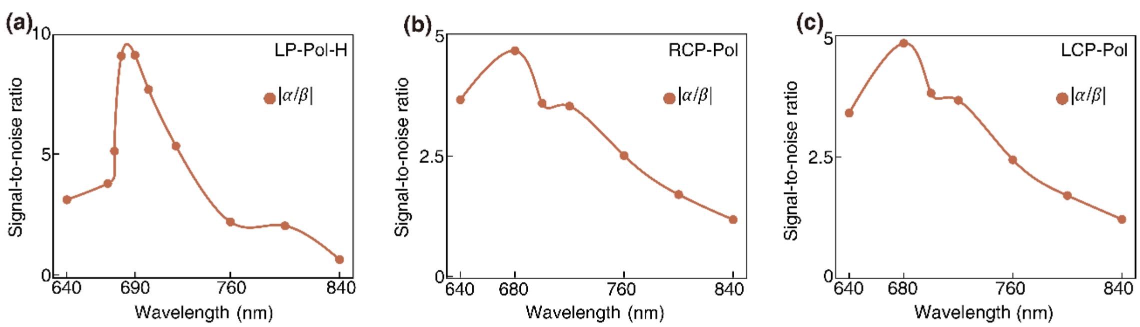

Fig. S3 The spectra of Jones matrix's signal-to-noise ratio of the metasurface polarizers. (a)–(c) Measured spectra of LP-Pol-H, RCP-Pol and LCP-Pol.

5 Jones matrix of the circular metasurface polarizers

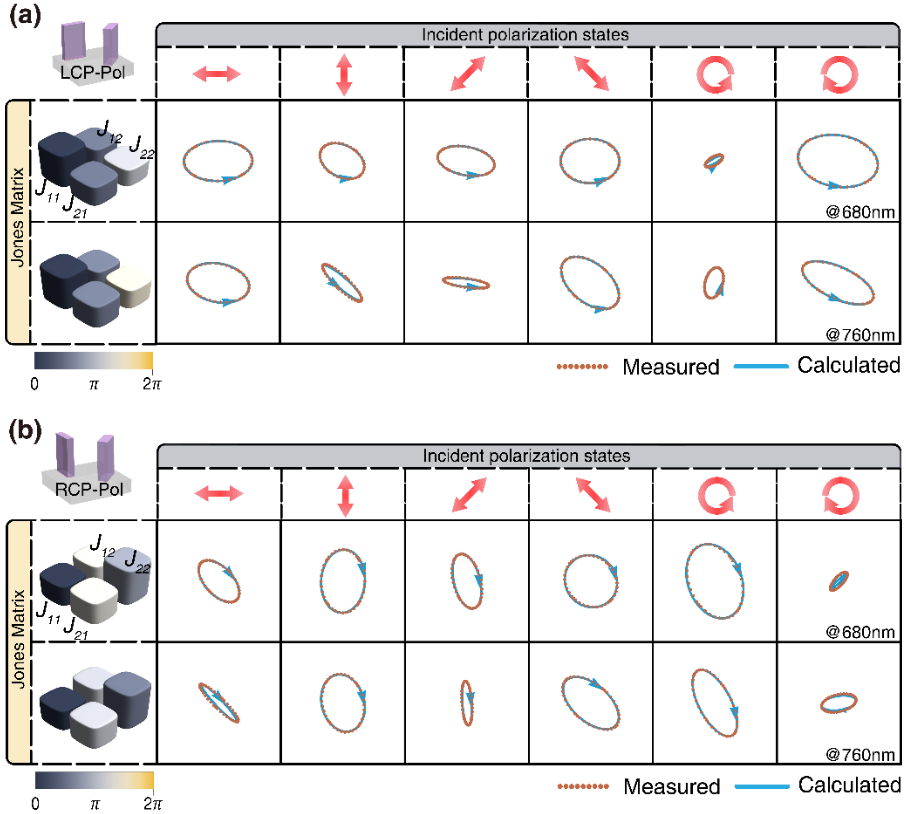

Fig. S4 Experimental retrieval of the Jones matrix of the circular metasurface polarizer. (a) Jones matrix of LCP-Pol. (b) Jones matrix of RCP-Pol. The polarization states of the incident light include linear polarizations along H, V, 45°, 135° directions, left- and right- circular polarizations (LCP and RCP). After passing through the metasurfaces, the polarization ellipses directions of the transmitted light at wavelengths of 680 nm and 760 nm were measured. The dot and solid lines correspond to the measured and calculated data. From the calculated results, we can obtain the spin direction of the electric fields which are indicated by the arrows. Each element of the Jones matrix is shown by the bar charts, in which the height and color are associated with the amplitude and phase.

6 Demonstration of the polarization image encryption

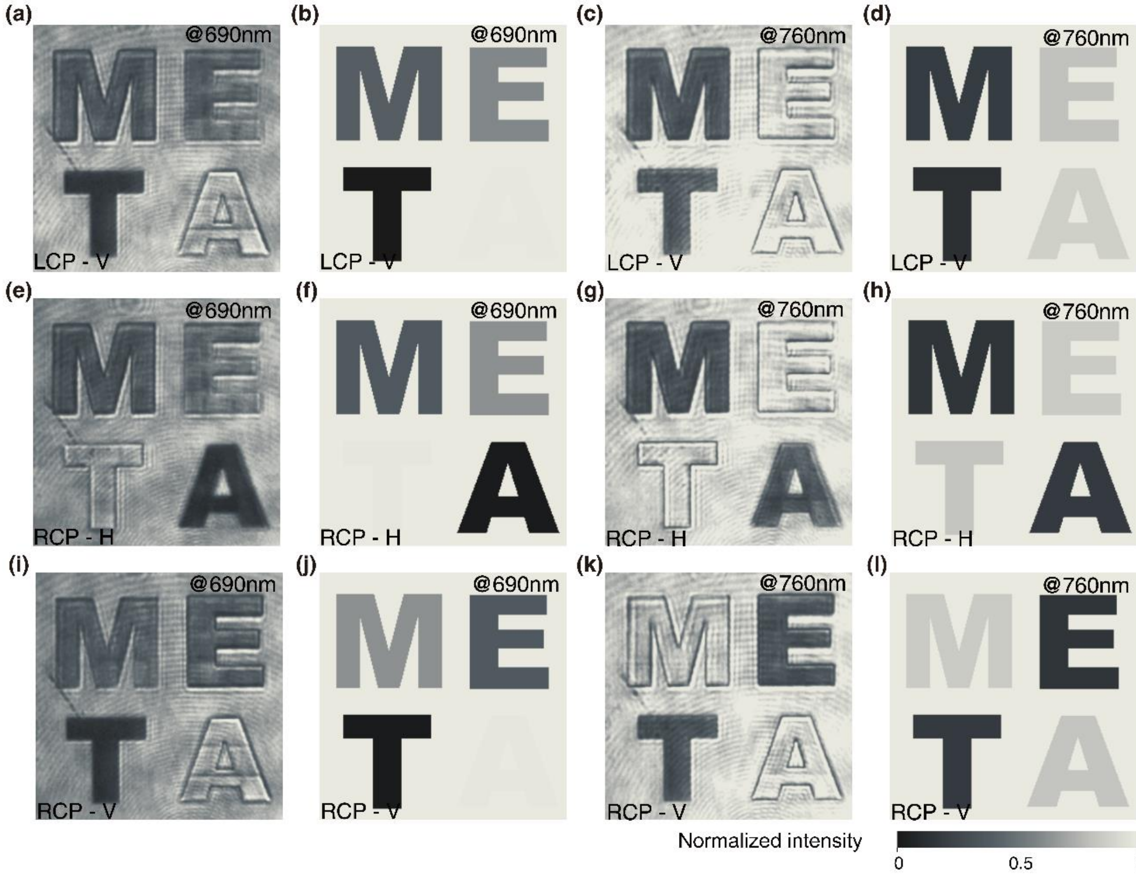

Fig. S5 Polarization image encryption with the linear metasurface polarizers. (a)–(l) Measured imaging results of polarization image encryption sample. For incident light with LCP and RCP states, the intensity distribution of the four letters M, E, T and A were measured at the wavelengths of 690 nm and 760 nm by using the vertical (a)–(d), (i)–(l) and horizontal (e)–(h) linear analyzers. The calculated intensity distribution of the four letters is based on the measured Jones matrix. The results are normalized to the maximum intensity values.

7 Optical holography with the circular metasurface polarizers

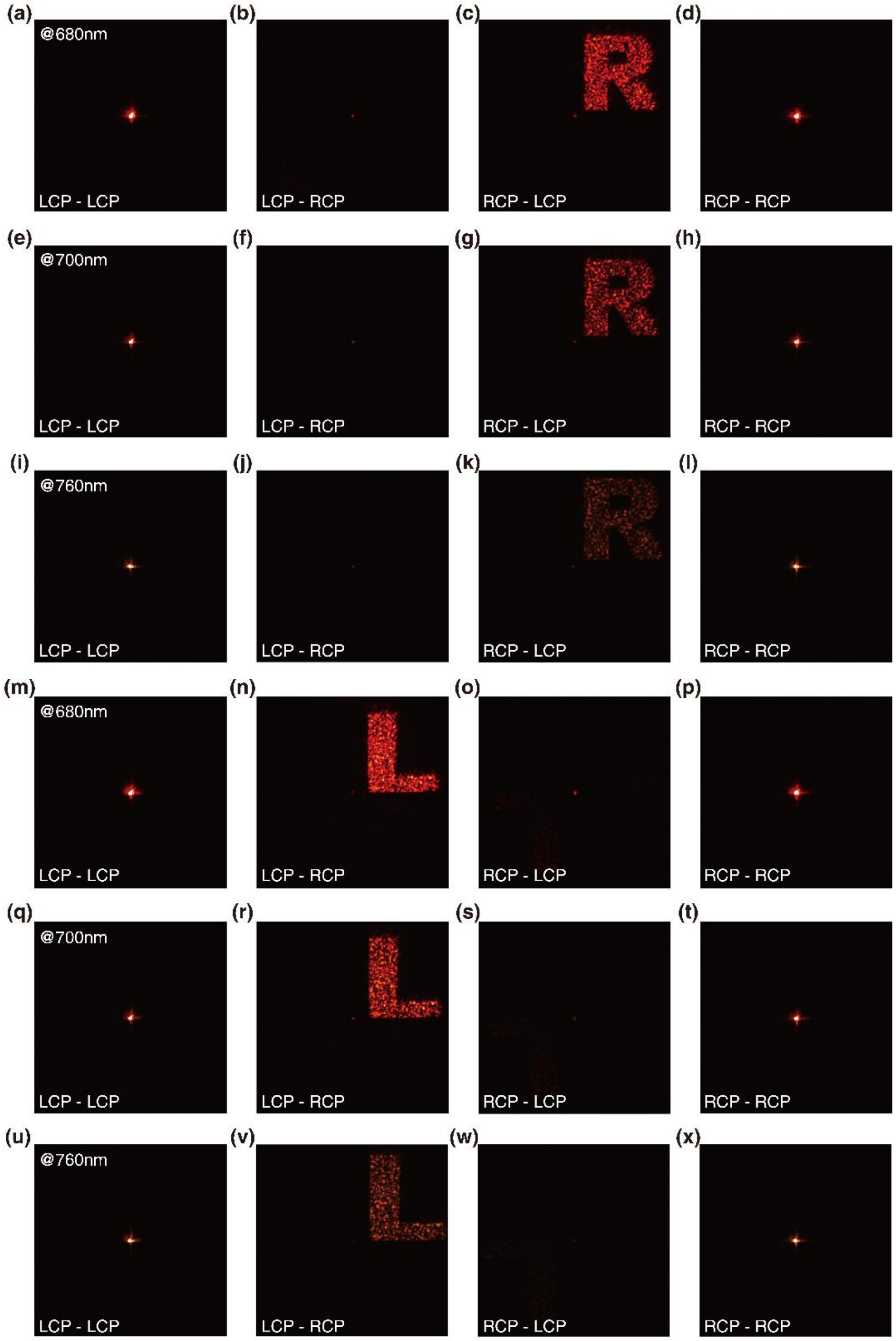

Fig. S6 The wavelength dependent holographic images of the RCP-Pol and LCP-Pol. (a)–(d) RCP-Pol at the wavelength of 680 nm. (e)–(h) RCP-Pol at 700 nm. (i)–(l) RCP-Pol at 760 nm. (m)–(p) LCP-Pol at 680 nm. (q)–(t) LCP-Pol at 700 nm. (u)–(x) LCP-Pol at 760 nm. Holographic images were obtained under the measurements of four polarizer and analyzer combinations.

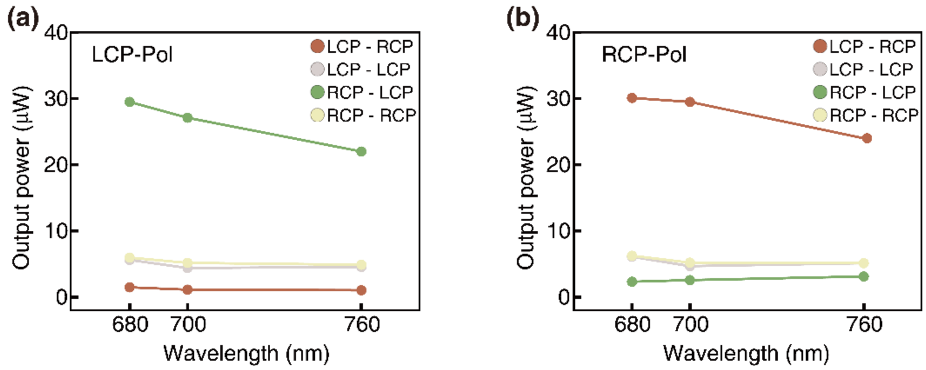

Fig. S7 Circular polarization resolved transmission efficiency of the metasurface holograms. (a) Measured spectra of the LCP-Pol metasurface hologram. (b) Measured spectra of the RCP-Pol metasurface hologram. The incident light has a power of 100 μW.